A STUDENT TEACHER'S SUPPORT FOR COLLECTIVE ARGUMENTATION

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We observed a student teacher's practice as she taught one unit of instruction. Using a modification of Toulmin's (1958/2003) diagrams, we analyzed her support for collective argumentation by examining questions she asked, parts of arguments she contributed, and the kinds of warrants her students contributed. Our findings suggest reasons she was identified as an exemplary student teacher and support moves that may be applicable to other teachers' practice.

As part of a larger study, we observed several student teachers as they each taught one unit of instruction. Our research team identified one of the student teachers, Bridgett (a pseudonym), as exhibiting practices more commonly seen in experienced teachers. Bridgett's facility for engaging in frequent, prolonged, and mathematically significant discourse with her students presented an opportunity for investigation into how collective argumentation might be supported by teachers. In this paper, we characterize her teaching moves in support of collective argumentation to answer the following question: In what ways did an exemplary student teacher support collective argumentation as she taught a geometry unit?

Background

Recent work in mathematics education has highlighted argumentation as an important part of classroom discourse. Building on Toulmin's (1958/2003) model of argumentation, mathematics educators, following Krummheuer (1995), examined collective argumentation in classroom settings. Collective argumentation involves people arriving at a conclusion, often by consensus. This avenue of research is an extension of Toulmin's work, as he examined argumentation in the traditional sense of one person convincing an audience of the validity of a claim. Some work in mathematics education examines individual construction of arguments (e.g., Hollebrands, Conner, & Smith, 2010; Inglis, Mejia-Ramos, & Simpson, 2007), but we build on studies addressing collective argumentation. Current work in collective argumentation involves examining student learning through this lens (Krummheuer, 2007) as well as examining how ideas become "taken as shared" (Rasmussen & Stephan, 2008).

An argument, as described by Toulmin (1958/2003) and currently used in the field, involves some combination of claims (statements whose validity is being established), data (support provided for the claims), warrants (statements that connect data with claims), rebuttals (statements describing circumstances under which the warrants would not be valid), qualifiers (statements describing the certainty with which a claim is made), and backings (usually unstated, dealing with the field in which the argument occurs). Toulmin conceptualized an argument as occurring with a specific structure (see Figure 1) in which these parts of arguments relate to one another in specific ways. In practice, arguments are often more complicated in that, for example,

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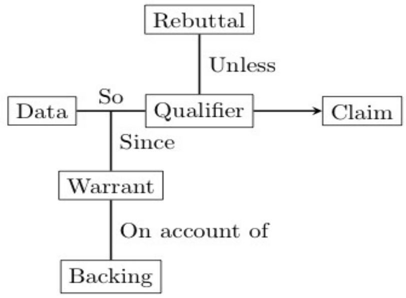
statements offered as data may also need support, thus functioning as both data in one argument



and a claim in a sub-argument[1].

Figure 1: Diagram of a Generic Argument (adapted from Toulmin, 1958/2003)

Our research answers Yackel's (2002) call to examine the teacher's role in collective argumentation by focusing on both the parts of arguments he or she provides and the teaching moves that prompt or respond to parts of arguments provided by students. Previous research has shown that facilitating mathematical discussions, of which collective argumentation is a part, is difficult for teachers, particularly when students have been engaged in tasks that may have multiple solution paths (Hufferd-Ackles, Fuson, & Sherin, 2004; Stein, Engle, Smith, & Hughes, 2008). Hufferd-Ackles et al. suggested creating a "math-talk learning community" (p. 81) in a classroom as one way to ameliorate the difficulties of facilitating productive discourse, while Stein et al. described five practices that are useful when facilitating discourse particularly around cognitively demanding tasks. Our research examines the role of one student teacher as she and her students engage in collective argumentation.

Methodology

We observed and videotaped one exemplary student teacher, Bridgett, for seven consecutive class periods, covering the bulk of a geometry unit within an integrated and accelerated freshman mathematics course. Members of the research team transcribed each of the video recordings. Field notes, class video recordings, and relevant written class materials supplemented the transcripts. We identified and diagrammed episodes of argumentation within the transcripts, purposefully ignoring non-mathematical classroom talk and arguments that were largely definitional (such as deciding how many sides a nonagon has) or pedagogical in nature. Individual or paired members of the research team created diagrams, and the entire team vetted each proposed diagram after intense discussion and frequent

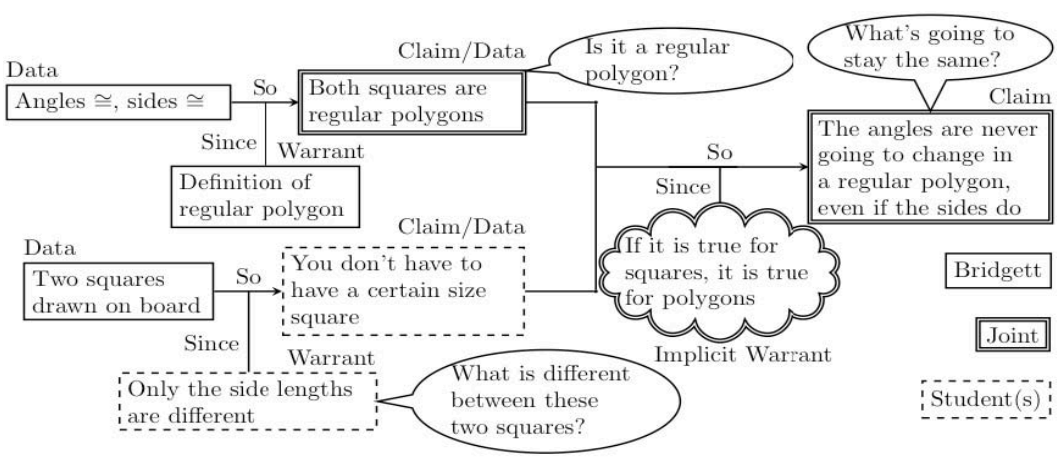


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Figure 2: Diagram of Argument about Regular Polygons

We used a modification of Toulmin's (1958/2003) diagrams to document the collective argumentation. We color coded[2] the diagrams to record whether Bridgett, the students, or both Bridgett and students together contributed a given component (as in Conner, 2008) and recorded Bridgett's actions in support of argument components. These practices allowed for a finer nuance in identifying who contributed a specific part of an argument and how that contribution came about. See Figure 2 for an example of Bridgett's support for argumentation. In addition, when a part of an argument was implied but not directly stated by the class, we included it in the diagram, labeling it as an implicit part. Most of these parts were warrants (see the "cloud" in Figure 2 for an example of an implicit warrant.

We categorized Bridgett's questions, other support and direct contributions across all the diagrams. We analyzed Bridgett's supportive acts in light of the structure of the arguments and their mathematical context. Our identified patterns of support constitute the core of the present results.

Results

Bridgett engaged in many different activities to support the collective argumentation in her class. She asked questions that prompted students to provide various parts of arguments, she responded to students' contributions in various ways (sometimes verbally acknowledging the contribution, sometimes recording the contribution on the board for future use), and she directly contributed parts of arguments. On a more global scale, she negotiated norms in her class that applied to all members that involved supporting claims with data and warrants. Our analysis of Bridgett's support for argumentation is ongoing; this report focuses on the parts of arguments prompted by Bridgett's questions, the particular kinds of warrants that were contributed by students, and the parts of arguments that Bridgett herself contributed. Through the analysis of these aspects of Bridgett's support for argumentation, we aim to provide a description of what is possible, even in a student teacher's classroom, and to describe the moves that Bridgett used that might be productive in other teachers' classes.

The episode of argumentation chosen to illustrate our points in two of the following sections (Figure 2) occurred during a whole class discussion about polygonal angle formulas. The class was engaged in an attempt to derive the formula for the measure of the interior angles of (regular) polygons. The class had already discussed various ways to partition particular polygons into triangles, in order to sum all the interior angles, and the observation was made that each angle in the polygon needed to be congruent if a general formula for the interior angle measures was to be found. Bridgett recognized that many of the students had only an intuitive understanding of regular polygons, and that the definitional properties of regularity were not explicit in some of the students' minds. Therefore, she explored with the class what should be true of "regular" polygons.

Questions as Support for Collective Argumentation

As a method to analyze Bridgett's support, our research team identified the various questions Bridgett posed that supported collective argumentation within the classroom. Examining her questions across the collection of arguments, we recorded 290 questions from Bridgett that directly supported a part of an argument. We then analyzed who contributed the various parts of the arguments and found that her questions prompted a total of 156 student contributions, 119 contributions from both the students and Bridgett, and 15 contributions from Bridgett. This first level of analysis reflects two important norms within Bridgett's support for collective

argumentation. First, Bridgett's support facilitated a significant amount of student contribution. Second, due to the significant amount of student contribution, Bridgett rarely needed to respond to her own questions; rather, the support she provided by asking appropriate questions elicited meaningful student participation. These findings seem to substantiate our assumption that classified Bridgett as an exemplary student teacher.

To further our analysis, we differentiated among the various parts of arguments prompted by Bridgett's questions. For example, her questions prompted more contributions of data than any other part of the argument. She asked 82 questions that prompted students to provide data, 67 that prompted data to which both Bridgett and her students contributed, and 6 that resulted in Bridgett herself providing data. Included in these numbers are parts that were coded as data alone and parts that were coded as both data and claim, as illustrated in Figure 2. The statement, "Both squares are regular polygons" was coded as both data and claim because it first was a claim, made by Bridgett and her students together, that the squares on the board were regular polygons. It was then used, together with "You don't have to have a certain size square" as data for the main claim in the argument. Although this large proportion of data supported by Bridgett's questions corresponds to the proportion of data provided within the arguments, this finding demonstrates how Bridgett's questions required students to provide the facts needed to support their development of mathematical claims. Bridgett's questions prompted many of the parts of arguments that were contributed, but not all contributions were prompted by questions. In the next section, we explore similarities and differences in student contributions of warrants prompted and unprompted by Bridgett.

Student Contributions of Warrants

Many of the questions posed by Bridgett supported students in making their reasoning explicit. Bridgett also supported students' reasoning in other ways, such as marking a diagram or encouraging the students to continue their thought processes. Of the 57 instances in which Bridgett provided any support for a warrant, 48 of the warrants were explicit and only 2 could be attributed solely to Bridgett. In contrast, of the 175 warrants that were unsupported, 63 warrants were explicit and Bridgett provided 15 of the explicit warrants. Thus, when Bridgett provided support for a warrant, students were more likely to make their reasoning explicit, either in collaboration with the teacher or on their own. However, when Bridgett did not provide support, the students were less likely to make their reasoning explicit, and Bridgett may have contributed the appropriate reasoning. These conclusions align with Toulmin's (1958/2003) suggestion that warrants are typically left implicit unless specifically requested, or, in our case, supported by the teacher. In addition, these conclusions speak to the importance of the teacher's role in collective argumentation; specifically, assisting students in making their reasoning available to the class.

Although it was typical for students to leave their reasoning implicit in the absence of support, there were 25 instances in which the students made their reasoning explicit without specific support from Bridgett. For this reason, we focused our analysis on the types of warrants provided by students with and without Bridgett's support. Our analysis yielded three categories of warrants: mathematical concepts (i.e., definitions, theorems, and properties), procedures, and observations. Of the 24 explicit warrants provided exclusively by students in conjunction with support from Bridgett, 8 were categorized as concepts, 12 were categorized as procedures, and 4 were categorized as observations. One example of an argumentation episode in which Bridgett provided support for a students' warrant was made while the class was reviewing homework. Bridgett posed the question to the class, "Is it possible to have a triangle with side lengths 3, 4, and 7?" The class responded, "No." Bridgett then asked, "Okay, why not?" One student

reasoned, "[Be]cause the two smallest sides have to be as long as the longest side. [Be]cause 3 plus 4 equals 7 and 7's not larger than 7." This argument is illustrated in Figure 3a. The student's warrant was categorized as concepts because the student's reasoning made use of the triangle inequality theorem although it was not precisely stated. Bridgett supported the warrant by asking the question, "Why not?"

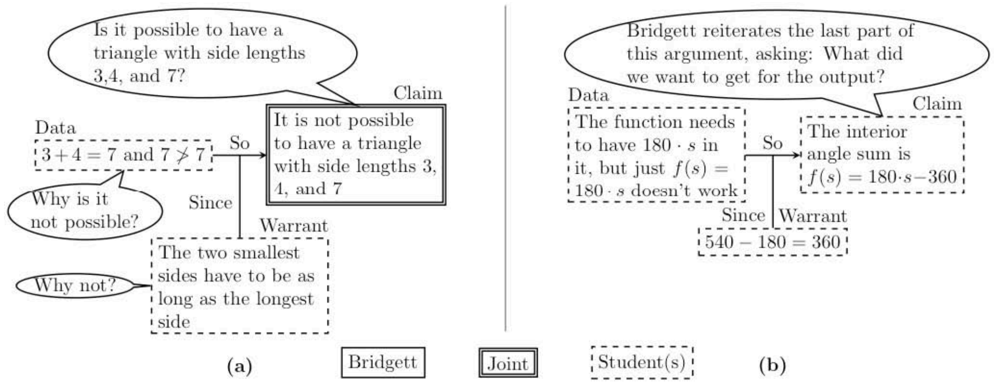

Figure 3: Arguments about (a) Triangle Side Lengths and (b) Polygon Angle Sum

In comparison, when students provide explicit warrants in the absence of support from Bridgett, 8 were categorized as concepts, 12 were categorized as procedures, and 5 were categorized as observations. For example, one episode (see Figure 3b) in which Bridgett did not provide support for a student's explicit warrant was made while the class determined the function for the sum of the measures of the interior angles of a polygon given the number of sides. A student was at the board explaining how he developed his formula. Claims that had entered into the collective up to this point include the linearity of the function, the slope of the function is 180, and 180(3) = 540 but the sum of the interior angles of a triangle is 180 degrees. The student claimed the function needed to have 180 as the slope, but 180 "doesn't work out right." He then stated, "But then I just subtracted 180 from 540 and it equals 360. Yeah. So, subtract 360." While making this statement, he wrote on the board $f(s) = 180s - 360$. The student's warrant was categorized as a procedure. Although Bridgett provided support for the claim, she did not provide support for his warrant.

Students provided the same types of warrants with the same approximate frequency, regardless of the presence of support. We draw two conclusions from this comparison. First, it appears that Bridgett negotiated classroom norms regarding the appropriate types of justification and provided support for warrants involving each of the types of reasoning. Second, the minimal amount of variation in the number of warrants in each category suggests Bridgett was not selective in the types of warrants she supported. Her choice of when to provide support may be due to many factors, including the complexity of the argument, the perceived mathematical abilities of the student(s) providing the claim, or the context of the argument. Further analysis is warranted and ongoing. Although we can only speculate as to why Bridgett may have chosen to provide support in particular circumstances, we can analyze the contributions she makes to an argument.

Bridgett's direct contributions

Bridgett directly contributed only 100 parts of arguments (out of more than 800 parts), slightly less than one per episode of argumentation, during the seven days included in this analysis. In addition, we attributed an additional 15 parts of arguments to her (coded as implicit contributions). Of her 100 explicit contributions, 59 were coded as data, with only seven of these

being coded as both data and claim. When a part of an argument serves as both data and claim, it is either presented as an intermediate claim that serves as data for a later claim, or it serves as a claim's support (data) that is later challenged. It is significant that most of Bridgett's data contributions were coded exclusively as data because it is indicative of the purposes they served in the arguments. In many cases, when Bridgett introduced a problem to the class, her contribution was coded as data because it included information necessary to solve the problem (and/or specified what the problem was). For instance, in the argument diagrammed in Figure 2, Bridgett started the episode of argumentation by asking, "What is different between these two squares?" after she had drawn two squares on the board. The two squares drawn on the board were coded as data because they were used in this way in the argument; they were attributed to Bridgett because she drew them. In fact, without these two squares, the argument would not have occurred. It is different in contribution than the student's contribution that is coded as claim/data. The student's claim/data used Bridgett's data to make a claim that was then used as data for the final claim. Thus, while Bridgett's drawings of squares initiated the argument, the crux of the argument depended on claims and data contributed by students.

The other data attributed to Bridgett in this argument was more aligned with the data provided by the students and was characteristic of the other kind of data provided by Bridgett. Bridgett pointed out that their definition required that all angles were congruent and all sides were congruent. This was not challenged by anyone. Perhaps in this simple case there was no possibility of a challenge, but in general, when Bridgett contributed a piece of data, there was no challenge by students that would provoke a sub-argument. The one major exception to this rule was an argument initiated by Bridgett after a student disagreed with her claim that a circle was not a polygon. In this one episode, in which almost all of Bridgett's data/claim contributions occurred, Bridgett engaged the class in a proof by contradiction, a proof method with which the students were not particularly familiar, and she was challenged at various points in the argument by her students. This episode, although different in several respects from typical classroom discourse in Bridgett's class, showed us several important things. First, students were not afraid to challenge Bridgett's statements, even though they chose not to do so under normal circumstances. Second, when the class was doing something that was clearly unfamiliar, Bridgett contributed more parts of the argument. Finally, norms in this class applied to both the teacher and the students. The students held Bridgett to the same standards of argumentation, in particular, justifying claims. This unusual episode shows the consistency of the kinds of data contributed by Bridgett. Bridgett's data remained unchallenged except when students were very unfamiliar with the method of argument and strongly disbelieved her claim.

Discussion and Conclusions

The expansion of Toulmin's (1958/2003) model provides researchers an effective tool for the examination of collective argumentation (Conner, 2008). Through the systematic dissection and classification of what often feels like "messy" discourse, we are able to more clearly discern patterns and recognize the contributions of the various actors in collective argumentation. Important considerations, such as by whom claims are made, who provides justification for those claims, and the extent of student and teacher contributions, are quantified, which allows for a more focused qualitative analysis. It is through this analysis method that we are able to pinpoint characteristics of Bridgett's teaching that suggest that it is exemplary.

Though defining "good teaching" is beyond the scope of this paper, Bridgett's student-centered classroom aligns with the view of effective mathematics teaching espoused by many

teacher preparation programs and policy documents (Wilson, Cooney, & Stinson, 2005). She is unusual in this regard, as previous studies have found that such practices, and the beliefs that drive those practices, often diminish once prospective teachers enter the classroom (e.g., Eggleton, 1995). Bridgett's emphasis on and willingness to flow with students' contributions greatly enriched the mathematical discourse in her classroom. The students eagerly participated in discussions by making claims and providing data, and they also demonstrated their acceptance of the resulting learning style by holding Bridgett to the same standard of justification by demanding warrants and providing rebuttals to claims they considered dubious. In short, Bridgett's teaching was exemplary from many points of view, especially in light of her status as a student teacher.

An important characteristic of good mathematics teaching is the ability to develop a classroom environment that engages students in doing mathematics (Wilson et al., 2005). Lampert and Cobb (2003) explained the importance of mathematical discourse within such an environment, "classrooms [can] not be silent places where each learner is privately engaged with ideas… [learners] need to talk or write in ways that expose their reasoning to one another and to their teacher" (p. 237). Lampert and Cobb emphasized the important role of the teacher in facilitating and supporting such productive mathematical discourse; however, the authors contend that much less is understood regarding the teacher's role. The case of Bridgett provides a glimpse into some of the important aspects of the teacher's role in supporting mathematical discourse through collective argumentation. Our detailed analysis of the collective argumentation within Bridgett's classroom introduces examples of teacher moves that promote student engagement in doing mathematics. The questions Bridgett posed as well as the additional support she provided encouraged her students to significantly contribute to the development of mathematical arguments. Therefore, Bridgett's teacher moves promoted collective argumentation in the truest sense of the word *collective*, for the entire classroom continually worked together to arrive at mathematical conclusions.

The foregoing analysis has revealed many aspects of Bridgett's mathematical discourse with her students. The types of data she provided and the situations in which she required warrants seem consistent with a high level of student-centered discourse. However, it is worth asking if the relationship is causal. Would mimicking Bridgett's argumentation style lead to similar outcomes for other teachers with other students? A definitive answer to this question is currently unknown, as there are several other characteristics that might reasonably have contributed to Bridgett's teaching style and subsequent classroom interaction. Two possible mechanisms not examined here, but included as part of the larger study whose final results are forthcoming, are Bridgett's belief structure (Cooney, Shealy, & Arvold, 1998) and her beliefs about mathematics, teaching, and proof (Conner, Edenfield, Gleason, & Ersoz, 2011). Nevertheless, Bridgett's propensity to request and expect warrants from her students, as well as her reluctance to directly provide intermediate data and claims in arguments, are practices that could conceivably enhance the mathematical discourse in other classrooms by encouraging fuller student participation, resulting in more authentic argumentation. Further research on the transferability, as well as the causality, of argumentation mechanics is needed in order to confirm or disprove this hypothesis.

Endnotes

1. Toulmin (1958/2003) called these "preliminary arguments" or "lemmas" (p. 90). We use sub-argument to indicate that these may not come before the other argument in terms of time and to avoid lemma, which has a specific mathematical meaning.

2. Color is represented by line style in diagrams in this paper.

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